Dressed state solution for a trapped two-level ion

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In a recent paper Moya-Cessa at al. [1] showed that the Hamiltonian of a two-level ion in a Paul trap under the effect of a laser field can be reduced to the model of Jaynes-Cummings without the rotating wave approximation that can be written as

$$H = \omega a^\dagger a + \frac{\omega_0}{2}\sigma_3 + g\sigma_1(a^\dagger + a) \tag{1}$$

being σ_1 and σ_3 the Pauli matrices, a and $a^\dagger$ the ladder operators and ω the frequency of the trap, ω_0 the level distance and g the coupling constant. Indeed, this latter model is somewhat ubiquitous in quantum optics and then, it is interesting to solve it in a regime where the coupling constant g with the laser field can be taken to be larger than the separation between the levels of the atom ω_0.

To accomplish this task we apply the method of dressed states as recently presented in [2] by the dual Dyson series [3]. This is done by taking as an unperturbed Hamiltonian

$$H = \omega a^\dagger a + g\sigma_1(a^\dagger + a) \tag{2}$$

and as a perturbation the atom Hamiltonian $\frac{\omega_0}{2}\sigma_3$. The dressed states are then written as eigenstates of (2)

$$|\psi_{n,\lambda}\rangle = e^{\frac{g\lambda}{\omega}(a-a^\dagger)}|n\rangle|\lambda\rangle \tag{3}$$

being $a^\dagger a|n\rangle = n|n\rangle$, $\sigma_1|\lambda\rangle = \lambda|\lambda\rangle$ and $\lambda = \pm 1$. The corresponding eigenvalues are $E_n = n\omega - \frac{g^2}{\omega}$. Thus, we have number displaced states [4].

It is seen that it is not enough to stop the analysis at the leading order given by the unitary evolution $U_F(t) = \sum_{n,\lambda} e^{-iE_n t}|\psi_{n,\lambda}\rangle\langle\psi_{n,\lambda}|$. Rather, as shown in [2] we compute the Hamiltonian $H_F = U_F^\dagger(t)\frac{\omega_0}{2}\sigma_3 U_F(t)$ to analyse the model to higher orders. In this way we are able to derive how the energy levels are shifted by the laser field and how they depend on the number of the photons involved in the interaction. Beside, the study of the transformed Hamiltonian H_F through the probability amplitudes permits to derive the Rabi frequency making here the rotating wave approximation, and the system is shown to flop between entangled states of displaced number states of the photon and dressed states of the ion. The resonance condition is given by the equality of a multiple of the laser frequency and the distance between the two modified energy levels of the ion.

In this way, when the ion is taken to be in its ground state and the field in the vacuum, as not all the amplitudes of the flopping are different from zero, the above non classical states can be generated.

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